

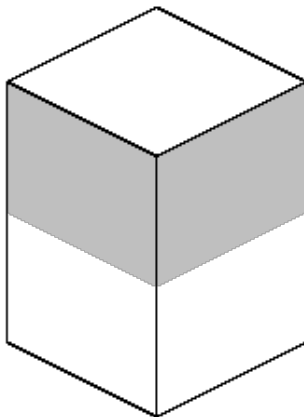
Exam Symmetry in Physics

Date April 1, 2025
Time 15:00 - 17:00
Lecturer D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are not allowed to use the lecture notes, nor other notes or books
- Use of a calculator is allowed
- All subquestions (a, b, c) of the three exercises have equal weight
- Illegible answers will be not be graded
- Good luck!

Exercise 1

Consider the colored square block depicted in the figure below:



The base of the block is a square. The rectangular sides all have the exact same pattern where the upper half is grey and the lower half is white.

- (a) Identify all symmetry transformations (rotations and reflections) that leave this block invariant and call the group that they form G_B .
- (b) Construct the character table of G_B and explain how the entries are obtained.
- (c) Show whether G_B allows for an invariant vector and/or an invariant axial-vector, or neither.

Exercise 2

Consider the group $O(3)$ of real orthogonal 3×3 matrices and its action on a rank-2 tensor σ_{ij} (i, j can take the values 1, 2, and 3).

(a) Show that any $O(3)$ -invariant tensor σ_{ij} satisfies $[D^V(g), \sigma] = 0$ for all $g \in O(3)$.

(b) Determine the subgroup of $O(3)$ transformations that leave the tensor

$$\sigma_{ij} = a\delta_{ij} + b\delta_{i3}\delta_{j3}$$

invariant, for nonzero constants a and b .

(c) Consider the subgroup C_{4v} of $O(3)$ transformations generated by c , a rotation around the z -axis, and b , a reflection in the y - z plane, for which:

$$D^V(c) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D^V(b) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Determine the invariant tensors this subgroup allows and check whether the result is in agreement with the answer obtained in subquestion (b).

Exercise 3

Consider the group $SU(2)$ of unitary 2×2 matrices with determinant equal to 1. Consider its action on the spin states $|s, m_s\rangle$ through the operator

$$U(\theta, \hat{n}) = \exp\left(\frac{i}{\hbar}\theta \hat{n} \cdot \vec{S}\right),$$

where \vec{S} denotes the spin operator.

(a) Write down the explicit matrix for S_z acting on the space of $|\frac{3}{2}, m_s\rangle$ states.

(b) Write down the explicit matrix for $U(\theta, \hat{n})$ acting on the space of $|\frac{3}{2}, m_s\rangle$ states for the specific case $\hat{n} = \hat{z}$, and determine the values of θ that describe all distinct $U(\theta, \hat{z})$.

(c) Explain why the $(2s + 1)$ -dimensional space spanned by $\{|s, s\rangle, \dots, |s, -s\rangle\}$ forms the carrier space of an irrep of $SU(2)$.